Does Price of an Essential Non-Renewable Resource Necessarily Grow?

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Dasgupta and Heal's 1974 paper extends Hotelling's 1931 partial equilibrium model into a dynamic general equilibrium model. Both papers show that nonrenewable resource prices do grow exponentially, which is called the Hotelling's rule in the literature. Empirical evidence on the contrary shows that most nonrenewable prices are constant in the long-run. The controversy between theory and empirical regulatory perhaps may be called the Hotelling's Paradox. This paper, based on Dasgupta and Heal (1974), shows that nonrenewable dependent growth does not always generate skyrocketing resource prices. In particular, this paper shows that resource price converges to a constant under Cobb-Douglas technology and that the model economy dies out under a particular value of elasticity of marginal utility.

Introduction

In his seminal article, Hotelling (1931) showed that price of a nonrenewable resource must grow at the real interest rate at optimum, a result known as the 'Hotelling's rule' in nonrenewable resource economics.[1] The Hotelling's rule is in partial equilibrium nature: a nonrenewable sector solves the dynamic problem of maximizing discounted profits over an infinite horizon, constrained by the initial stock of the nonrenewable. The dynamic profit maximization problem yields that the resource price must grow at the (real) rate of interest, the discounting factor, when extraction is costless and the market is perfectly competitive. Since interest rate is taken constant in a partial equilibrium solution, resource price grows infinitely.

Professors P. Dasgupta and G. Heal overcome partial equilibrium nature of the rule by publishing their seminal paper "the optimal depletion of exhaustible resources" in 1974. In their paper, Dasgupta and Heal (1974), henceforth D-H, integrated a final-good market with a nonrenewable resource market in a Ramsey setup, and analyzed the optimal depletion of nonrenewable resources. One unfortunate assumption made in their paper however precluded D-H to unveil the distinguishing characteristic of general equilibrium version of the rule from its partial equilibrium version. In particular, D-H ignored differentiating between the rental rate of capital and the interest rate. This was a critical misspecification because a growth setup with a depleting resource as an input in final-good production technology has a peculiar characteristic that the time paths of rental rate of capital and resource price can be determined by the resource sector's efficiency condition independently from the rest of the model. This happens because resource price and rental rate of capital can be expressed in terms of their respective marginal productivities, which can be further defined in terms of capital/resource extraction ratio. Hence, mathematically speaking, when rental rate of capital and interest rate are assumed to be one and same thing, it allows the differential equation derived from resource sector efficiency condition grow infinitely in terms of capital/resource extraction ratio (which can be expressed also in terms of rental rate of capital or resource price). Clearly, the distortion is fed back to resource price and rental rate of capital, and hence these results also become distorted. In the final analysis, the partial equilibrium version of the Hotelling's rule is reproduced by Dasgupta and Heal (1974).

It has been well documented that the Hotelling's rule is not supported by the empirical regularity. Looking at real price behaviors of several nonrenewables like aluminum, copper, iron ore, lead, silver, and tin prove that there is no apparent trend in rate of growth of resource prices in the last century.[2] Given the (existing) theory, the reaction to this paradox in the literature has been twofold: (i) incorporating additional elements into the partial equilibrium model such as exploration costs, capital investment and capacity constraints, ore quality variations, (ii) modifying econometric techniques and/ or data.

[1] A good exposure to this literature can be attained by the following studies. First, Gordon (1967) can be read to get a good sense of the pre-Dasgupta and Heal (1974) literature. Smith (1968) can be also useful in that respect. Second, Dasgupta and Heal (1974) and Stiglitz (1974a, 1974b) must be read as they incorporate a nonrenewable resource sector into a growth framework. Third, surveys of Peterson and Fisher (1977) and Krautkraemer (1988) can be read for a good exposure to the post-Dasgupta and Heal literature.
[2] Interested readers may find rich data at http://minerals.usgs.gov/minerals/pubs/of01-006/ on nonrenewable prices. See also e.g., Krautkraemer (1998).

Nobody ever however, to our knowledge, has ever questioned the partial equilibrium nature of the Hotelling (1931) or misspecification in Dasgupta and Heal (1974).

This study presents the true general equilibrium version of the Hotelling's rule and shows that resource prices will converge to a constant under Cobb-Douglas (henceforth, C-D) technology.[3] In that respect, this study argues that the paradox between the theory and empirics may indeed be a fictitious one. The second contribution of this study is the advancement towards a full analytical solution of the D-H setup. This study presents the complete analytical solution under a specific elasticity of marginal utility assumption. We know that assigning a particular value to elasticity of marginal utility may distort the behavior of real economy (physical capital, output, and the nonrenewable resource). Nonetheless, having a complete analytical solution still generates useful information to reveal rules for sustainable consumption and growth.

A short summary of the model is as follows. Two factors of production, a reproducible capital and a nonrenewable resource, are used to produce a final output, which can be consumed or invested. We assume the final-good production technology is C-D, which implies that both factors are essential. Our main motivation for relying on C-D technology is analytical tractability. Profit-maximizing firms operating in the good market imply a unique resource price/rental ratio and a corresponding optimal capital/resource ratio. A nonrenewable resource extracting sector solves the dynamic problem of maximizing discounted profits over an infinite horizon, constrained by the initial stock of the nonrenewable. The main difference between D-H and our approach appears to be our differentiation between rental rate of capital and of loans. Our results diverge substantially by this single assumption, nonetheless. In particular, we demonstrate that the price of the nonrenewable does not grow in the long-run, albeit the resource is essential in production. Our explanation to this finding is as follows: Economic agents with perfect foresight in a depleting economy recognize that there is no way to sustain consumption and production in the long-run, when a depleting resource is essential. In such a case, optimal behavior is to plan a smooth depletion of an economy instead of accepting rising prices of the nonrenewable, just to sustain consumption and production for a while. Hence, contrary to what Hotelling's rule suggests, it is optimal to observe resource prices increasing at decreasing rates or decreasing at decreasing rates, depending on the ratio of initial extraction to initial capital stock. That is why we observe constant resource price in the long-run.

The organization of the paper is as follows. The second section presents the basic model and the simulations under a specific elasticity of marginal utility value. We show that the real economy peters out in time under this constraint. The fourth section presents concluding remarks.

[3] Heuristically speaking, we may argue that similar results would hold for a CES technology with inelastic substitution, due to Gaitan, Tol, and Yetkiner (2004), henceforth G-T-Y. Though GTY (2004) is developed under exogenous saving assumption, results would not change in the long-run. It must be noted that the genuine weakness of such an assumption is that the (critical) role played by the impatience of economic agents on depletion and on the rest of the economy is ignored.

The Model

We assume that total consumer utility of a representative consumer consists of the present value of an infinitely long stream of consumption of final output, as given by the standard constant intertemporal elasticity of substitution utility function:

$$U = \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt \qquad \theta, \rho > 0 \tag{1}$$

In equation (1), U represents total utility, $C(t)$ is the flow of consumption at time t, ρ is the subjective rate of discount, and $1/\theta$ is the elasticity of substitution between flows of consumption at different points in time. For simplicity, we assume that the family size of the household does not change in time. Households hold assets in the form of ownership claims on capital and nonrenewables. Households are competitive in that each takes as given the real rental rate of capital $r(t)$ and the real price of nonrenewable resource $q(t)$. The flow budget constraint for the household is

$$\dot{A} = q(t) \cdot R(t) + \left(r(t) - \delta\right) \cdot A(t) - C(t) \tag{2}$$

In (2), $r(t) - \delta \equiv i$ is the real rate of interest, and $r(t)$ is the rental rate of capital. The equation states that assets rise with income $q(t) \cdot R(t) + i(t) \cdot A(t)$ and fall with consumption $C(t)$.

The household's optimization problem is to maximize U in equation (1), subject to the budget constraint in equation (2), the stock of initial assets $A(0) = A_0$, and the transversality condition (see below). As the momentary utility function satisfies the Inada (1963) condition that the marginal utility of consumption becomes infinite when $C(t) \to 0$, the inequality restriction $C(t) \geq 0$ does not apply. The present-value Hamiltonian is

$$J = e^{-\rho t} \frac{C^{1-\theta} - 1}{1-\theta} + \mu\left\{qR + (r - \delta)A - C\right\} \tag{3}$$

We omit time subscripts in the subsequent analysis whenever no ambiguity results. The variable μ is the present-value shadow price of income. It represents the value of an increment of income received at time t in units of utils at time 0. The first order conditions for a maximum of U, in addition to (2), are

$$\frac{\partial J}{\partial C} = 0 \Rightarrow e^{-\rho t} C^{-\theta} = \mu \tag{4}$$

$$\dot{\mu} = -\frac{\partial J}{\partial A} \Rightarrow \dot{\mu} = -\mu\{r - \delta\} \tag{5}$$

where the transversality condition is $\lim_{t \to \infty} [\mu(t) \cdot A(t)] = 0$. The reduced form of the canonical system turns out to be

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\{r - \delta - \rho\} \tag{6a}$$

$$\dot{K} = qR + (r - \delta)K - C \tag{6b}$$

under the assumption that total assets A is equal to total capital stock K, which must hold in a closed economy without government. This completes the household side of the model. Let us turn to the production and extraction sides now.

We assume that output Y is produced by using physical capital K and a nonrenewable resource R. Production technology is represented by $Y = K^{\alpha}R^{1-\alpha}$, where α is production elasticity of capital.[4] Notably, $F(\bullet)$ is increasing, strictly concave, twice differentiable, and homogenous of degree one. Each input is essential under C-D technology, and the Inada conditions guarantee that $K > 0$ and $R > 0$. Under a perfectly competitive market assumption each factor will be paid (in real terms) its marginal contribution to production:

$$F_K = r \tag{7a}$$

$$F_R = q \tag{7b}$$

In (7), r and q are, respectively, the rental price for a unit of capital services and the nonrenewable resource price, and F_K and F_R are the marginal productivities of K and R. This constitutes the production side of the economy. Let us now consider the extraction sector.

Suppose that the resource market is a perfectly competitive one (hence, prices are given) and that extraction is costless. Under these assumptions, the representative firm would solve the following maximization problem:

$$\begin{aligned} &Max \quad \int_0^{\infty} [qR] e^{-\int_0^t i(\tau)d\tau} dt \\ &s.t. \quad \int_0^{\infty} R(t)dt \le S_0 \end{aligned} \tag{8}$$

In (8), i is the (endogenously determined) real interest rate, S_0 is the initial stock of the nonrenewable, and $i \equiv r - \delta$. Equation (8) is an *isoperimetric problem* of calculus of variations. The *Lagrangian* integrand becomes

$$\Gamma = q\,\mathrm{R}e^{-\int_0^t (r(\tau)-\delta)d\tau} - \lambda R \tag{9}$$

[4] A relatively broad production function specification would be a CES technology such as $Y = \left(\alpha K^{\sigma} + (1-\alpha)R^{\sigma}\right)^{1/\sigma}$, where α is the *distribution parameter* and $\varepsilon = 1/(1-\sigma)$ is the *elasticity of substitution* between K and R. Unfortunately, it is *not* possible to derive an analytical solution under the CES specification. Furthermore, we know since G-T-Y (2004) that the case that factors of production are complements yields similar results to CD and that the case in which factors of production are substitutes yields results analogous to what D-H found. Therefore, exploiting the CD form to derive, to solve, and to study the dynamic economic system is sufficient.

where λ is *Lagrange multiplier* and *constant* by definition, and the respective transversality condition is $\lim_{t\to\infty} q\,\mathrm{Re}^{-\int_0^t (r(\tau)-\delta)d\tau} = 0$. The solution of the isoperimetric calculus of variations problem leads to:

$$q(t) = \lambda e^{\int_0^t (r(\tau)-\delta)d\tau}. \tag{10}$$

Equation (10) is a non-arbitrage condition saying that the nonrenewable is essentially an asset and therefore its (real) price must increase at the real interest rate:

$$\dot{q}/q = r - \delta \tag{11}$$

Equation (11) is the well-known Hotelling's rule in its simplest form.[5] Recall that in partial equilibrium r is presumed constant, and therefore q grows at the constant rate $r-\delta$. In our model, r is determined endogenously. In D-H, r is also determined endogenously. The difference between our model and their model arises from the fact that interest rate i and rental rate of capital r are identical in their model, which implies an unconstrained growth on physical capital/resource extraction rate. Therefore, their results reproduce partial equilibrium result of the Hotelling's rule.

The solution procedure of the model is as follows. Firstly, from (7), it is straightforward to show that

$$r(t) = \alpha\left(\frac{1-\alpha}{q}\right)^{\frac{1-\alpha}{\alpha}} \tag{12}$$

Using this information in equation (11) leads to the following differential equation:

$$\dot{q} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} q^{1-\frac{1-\alpha}{\alpha}} - \delta q \tag{13}$$

This is an ordinary non-linear differential equation. Its solution is:

$$q(t) = \left(\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} + \left((q(0))^{1-\alpha} - \frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} \right) * e^{-\frac{(1-\alpha)}{\alpha}\delta \cdot t} \right)^{\alpha/(1-\alpha)} \tag{14}$$

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[5] Notably, this is exactly what D-H find in their paper, $\frac{\partial F_R}{\partial t}\frac{1}{F_R} = F_K - \delta$ (obviously, their model does not include capital depreciation).

Remarkably, the initial value of q, $q(0)$, provides critical information in determining the exact path of q. If $(q(0))^{(1-\alpha)} > \left[\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}\right]$ ($(q(0))^{(1-\alpha)} < \left[\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}\right]$), then the $q(t)$ path converges to $\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}$ from above (below). If $(q(0))^{(1-\alpha)} = \left[\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}\right]$, then the $q(t)$ path is constant. We remind the reader that the $q(t)$ path approaches infinity in D-H due to the fact that their version of (13) does not discriminate between rental rate of capital and interest rate (see Annex B for the complete derivation of D-H model).

Using this solution, it is straightforward to determine the time path of $r(t)$:

$$r(t) = \alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} + \left((q(0))^{1-\alpha} - \frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} \right) * e^{-\frac{(1-\alpha)}{\alpha}\delta \cdot t} \right)^{-1} \qquad (15)$$

Observe that rental rate of capital converges to $r^* = \delta$ and price of nonrenewable to $q^* = (1-\alpha)\left(\frac{\alpha}{\delta}\right)^{\alpha/(1-\alpha)}$, irrespective of whether q approaches its steady state from above or below.

Some comments are in order due to (14). Firstly, the fact that nonrenewable price converges to a constant in the model is a substantial divergence from what the Hotelling's rule implies. Secondly, though the price of nonrenewable converges to a constant in the steady state, it does not imply that all nonrenewables and all countries (representing closed economies) have identical dynamics. Suppose that there are two completely closed countries, owning the same resource but have substantially different $q(0)$ values (which are function of elasticity of marginal utility and subjective rate of discount as much as technology parameters). In that case, the same resource will show different price dynamics in different economies. Thirdly, elasticity of marginal utility θ and subjective rate of discount ρ have no effect on the time path of q and r (though they have in $q(0)$, $r(0)$, K and R). In other words, q and r are independent of household's time-patience and desire for consumption smoothness preferences, except their initial values. This result is due to the fact that the Hotelling's efficiency condition imposes a solution for q and r independent from the rest of the model.

The rest of the model can be solved as follows. First, from (6a)

$$C(t) = \left(\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} + \left((q(0))^{\frac{1-\alpha}{\alpha}} - \frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} \right) * e^{-\frac{1-\alpha}{\alpha}\delta t} \right)^{\frac{\alpha}{(1-\alpha)\theta}} e^{-\frac{\rho}{\theta}t} (q(0))^{-\frac{1}{\theta}} c(0) \quad (16)$$

where $c(0)$ is a constant. The most interesting characteristic of consumption path is the finding that consumption peters out and approaches zero in time, irrespective of parameter values and initial value. Secondly, when consumers are very concerned with smoothing consumption over time or are very patient, the fall in consumption slows down. Finally, the fact that consumption peters out in time implies that utility is bounded from above irrespective of parameter-value assumptions.[6]

Second, we may simplify the differential equation in (6b) as follows:[7]

$$\dot{K} - \left(\frac{r(t)}{\alpha} - \delta \right) K = -C(t) \tag{17}$$

A first step in solving (17) is to note that it is a linear first-order differential equation.[8] Hence, multiplying both sides of (18) by the *integrating factor* $\pi(t) = \left(\left(\frac{\alpha(1-\alpha)}{\delta} \right)^{\frac{(1-\alpha)}{\alpha}} + \left((q(0))^{1-\alpha} - \left(\frac{\alpha(1-\alpha)}{\delta} \right)^{\frac{(1-\alpha)}{\alpha}} \right) e^{-\frac{1-\alpha}{\alpha}\delta t} \right)^{\frac{-1}{(1-\alpha)}} e^{-\frac{1-\alpha}{\alpha}\delta t}$ allows us to write (17) as follows

$$\pi(t)K(t) = -\int \left[\left(\left(\frac{\alpha(1-\alpha)}{\delta} \right)^{\frac{(1-\alpha)}{\alpha}} + \left((q(0))^{1-\alpha} - \left(\frac{\alpha(1-\alpha)}{\delta} \right)^{\frac{(1-\alpha)}{\alpha}} \right) * e^{-\frac{1-\alpha}{\alpha}\delta t} \right)^{\frac{\alpha-\theta}{(1-\alpha)\theta}} e^{-\frac{\rho}{\theta}t} e^{-\frac{1-\alpha}{\alpha}\delta t} c(0) \right] dt + c_2 \tag{18}$$

In (18), c_2 is a constant that comes out from integration. We will drop it from the equation as its value must be zero for satisfying the transversality condition. To our knowledge, it is not possible to solve the integration problem on the RHS of (18), unless a specific value for θ is assumed. It should be noted that a restriction on θ may change time behavior of K and R but not of q, r, and C (except their initial values). We will assume that $\theta = \frac{\alpha}{2-\alpha}$ in (18). By this assumption, we force θ to be $0 < \theta < \alpha < 1$ and that the household does not have a strong preference for a smooth consumption pattern. Hence, a simulation of consumption path would show a hump-shaped pattern with a very steep decay towards zero.

Under the specific θ assumption, the power of the first term in the integrand becomes unity and hence solution becomes possible. In particular, we find the capital path as

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[6] Checking for bounded utility is not algebraically straightforward. By intuition, nevertheless, we know that utility is bounded from above because consumption is declining in time. This result can be more clearly seen for, say, $\theta = \alpha/(2-\alpha)$ assumption.

[7] Use equation (7a) to see this.

[8] See Annex A for the derivations.

$$K(t)=\frac{a\cdot(q(0))^{-\frac{1}{\theta}}c(0)}{\left[\frac{\rho}{\theta}+\frac{(1-\alpha)\delta}{\alpha}\right]}\left(a+b*e^{-\frac{1-\alpha}{\alpha}\delta t}\right)^{\frac{1}{(1-\alpha)}}e^{-\frac{\rho}{\theta}t}$$
$$+\frac{a\cdot(q(0))^{-\frac{1}{\theta}}c(0)}{\left[\frac{\rho}{\theta}+\frac{2(1-\alpha)\delta}{\alpha}\right]}\left(a+b*e^{-\frac{1-\alpha}{\alpha}\delta t}\right)^{\frac{1}{(1-\alpha)}}e^{-\frac{\rho}{\theta}t}e^{-\frac{(1-\alpha)\delta}{\alpha}t} \tag{19}$$

In (19), $a=\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}$ and $b=\left((q(0))^{1-\alpha}-\frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}\right)$. Notably, capital stock converges to zero at the limit, $\lim_{t\to\infty}K(t)=0$. The faster the petering out of capital stock, the more impatient the consumers are.

The respective path of nonrenewable resource follows from (19) and (7):

$$R(t)=\left(\frac{r(t)}{\alpha}\right)^{\frac{1}{1-\alpha}}K(t) \tag{20}$$

Obviously, nonrenewable resource depletes smoothly towards zero in time. Again, the petering out of nonrenewable is faster, the more impatient the consumers. Notably, the transversality condition of problem (9), $\lim_{t\to\infty}qRe^{-\int_0^t(r(\tau)-\delta)d\tau}=0$, is satisfied. Finally, $R(0)$ and $C(0)$ are determined via (19) and the resource constraint equation in (8), that is, $S_0=\int_0^\infty R(t)dt$, for $K(0)=K_0$ (see Annex A). Hence, we derive the complete analytical solution.

The utmost important finding of our derivations is that resource price q is *constant* in the long run under a C-D production technology. Mathematically speaking, this is due to the fact real rate of interest $r-\delta$ is approaching zero at the steady state. Economically speaking, this result is due to the fact that economic agents with perfect foresight find a smooth depletion of resource and economy optimal. This is in sharp contradiction with the prediction of partial Hotelling's rule that an economy will "resist" depletion by offering increasingly higher prices to the resource to enhance higher consumption and production temporarily.

Stability

Equations (6a), (6b), and (11) form the canonical system in the model. Equation (11) can be solved separately. It directly implies a solution for (6a). Consequently, (6b) can be solved. Hence, the stability of the canonical system is indeed defined by (11). It is straightforward to check the stability of (11) and show that it is indeed stable.

Monopoly

An alternative market structure assumption in the resource market is monopoly. In our model, a monopolist who owns the only deposit takes into account the relationship between q and R, so that the necessary condition in (10) becomes marginal revenue equal to marginal user cost. Hence, marginal revenue (and not price) will rise at the rate of interest (in case of zero extraction costs). But this in itself does not tell us whether the resource will be extracted more or less rapidly than by competitive producers. Some, following Hotelling (1931, p.153), might assume that the rate of resource extraction is reduced because of "the general tendency for production to be retarded under monopoly". However, as Weinstein and Zeckhauser (1975), Sweeney (1977), Stiglitz (1976), and Kay and Mirrlees (1975) discussed and showed, the deviation in the extraction behavior of monopolist with respect to the perfectly competitive case depends on the price elasticity of demand. In particular, under the constant elasticity demand schedules, with zero extraction costs, monopoly prices and competitive equilibrium prices will in fact be identical, and hence the rate of utilization of the resource. Since our analytical model exploits a Cobb-Douglas technology, it implies a constant elasticity demand and therefore monopoly and perfectly competitive cases are identical.

A Tabular Summary of Results

We present at Annex B the full solution of D-H. A comparison of results is as follows:

Table 1. Comparison of Steady-states

Variable	D-H (δ=0)	Yetkiner (δ>0)
r	0	δ
q	∞	$(1-\alpha)(\alpha/\delta)^{\alpha/(1-\alpha)}$
K	0	0
R	0	0
C	0	0
$\chi = K/R$	∞	$(\alpha/\delta)^{1/(1-\alpha)}$

As Annex B and Table 1 reveals, the D-H assumption that rental rate of capital and interest rate are equal forces capital-extraction ratio χ to grow infinitely. Since r and q (and C) are solely function of χ, it also results in distorted factor prices in the model. Our market solution of the same problem shows that indeed resource price converges to a constant in the long-run. When we consider technology in the model, we find that the model-economy can generate sustainable consumption within the system.

Simulations

In this subsection, we will compare the three solutions via simulations. The advantage of this approach to the tabular summary is the fact that we fully exploit the power having complete analytical solution by also comparing transitional dynamics of the three cases. Let us assume that $\alpha = 0.5$, $K_0 = 1000$, $S_0 = 1000$, $\delta = 0.1$, $\rho = 0.02$, $\theta = \alpha/(2-\alpha) = 0.6$. It must be noted that we need to take up high values of K_0 due to the fact that the economy is depleting. Other parameter-value assumptions are quite common in the literature. The initial step on simulations is to determine $C(0)$ and $R(0)$. In the Appendix, we indicate how they are calculated.

Figure 1a: Time path of resource price

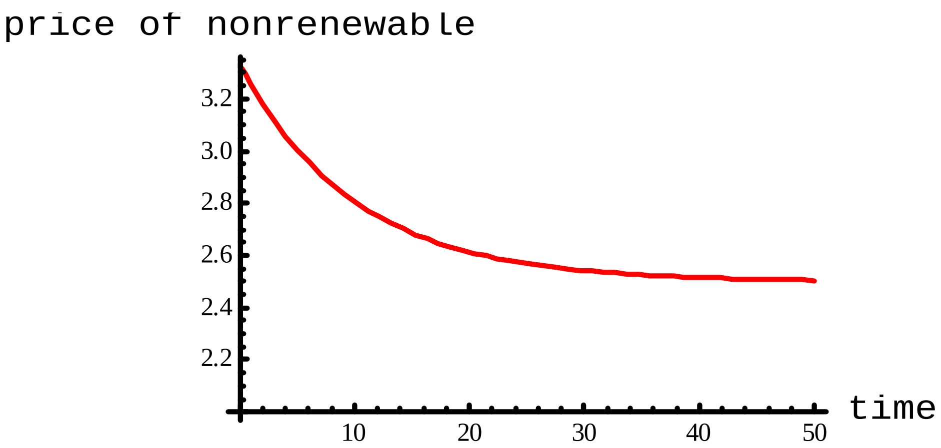


Figure 1b: Time path of rental rate of capital

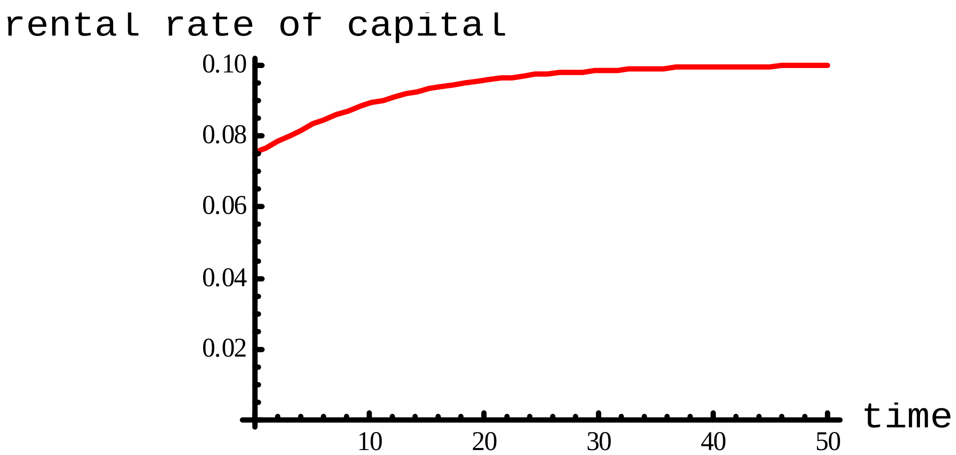


Figure 1c: Time path of consumption

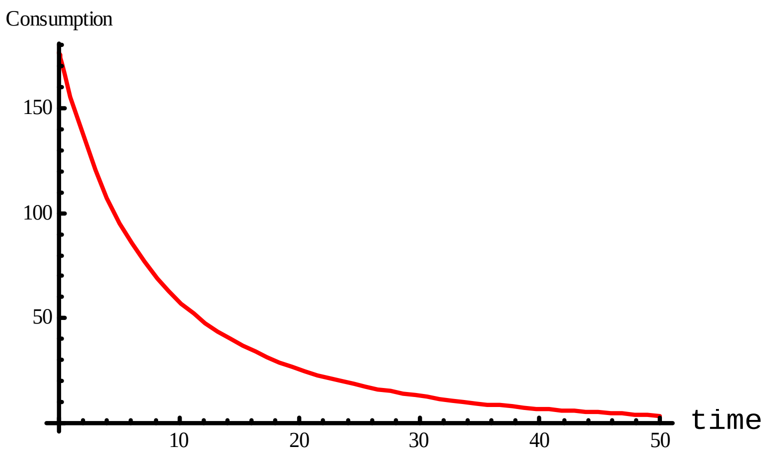


Figure 1d: Time path of capital

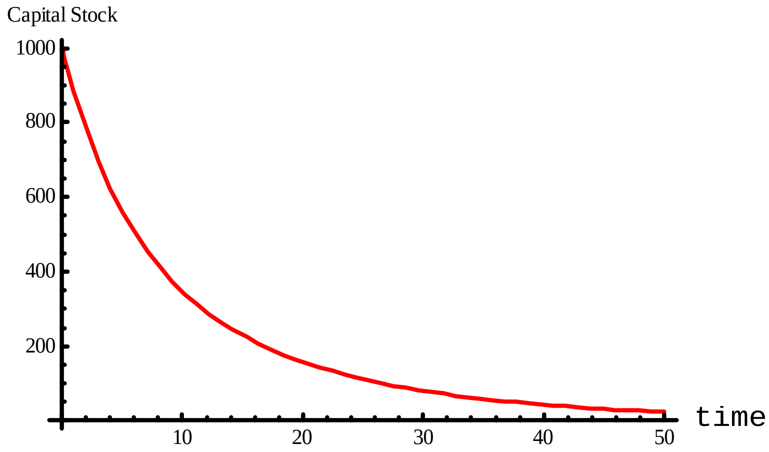
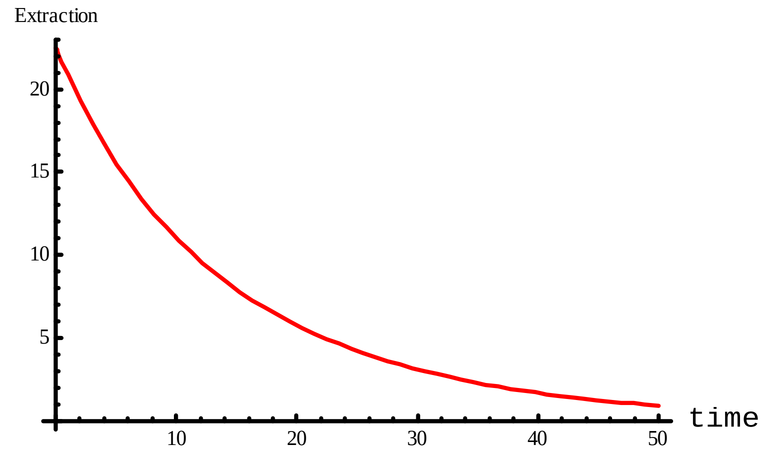


Figure 1e: Time path of resource extraction



The clear impact of the true formulation of the Hotelling's Equation is seen on the behavior of resource price, when we compare D-H and our results. We observe that resource price is indeed converging to a constant in the long-run, contrary to what D-H suggests. A second observation about the difference between D-H and our results is that D-H model show non-smooth (hump-shaped) behavior in consumption, capital and resource depletion, which we believe is due to the fact that the model is forced to offer rising prices to the resource that disturbs the optimal behavior of aforementioned quantities.

Conclusion

In this paper we studied the growth impact of a nonrenewable resource in a Ramsey framework. Our study is inspired very much by the seminal paper of Dasgupta and Heal (1974). The main difference between our model and their model is that we differentiate between rental rate of capital and interest rate in the model. The distinguishing property of a growth model with an essential depleting resource in production is that resource price, rental rate of capital and consumption are determined independently from the rest of the model. Since, Dasgupta and Heal's approach forces capital extraction ratio to grow infinitely, the approach also distorts the input price movements and reproduces partial equilibrium Hotelling's rule. Our approach on the other hand showed that the price of the nonrenewable converges to a constant in the long run under the Cobb Douglas case.

Indeed, we find this result quite intuitive under the perfect foresight assumption. If an economy has to decline sooner or later, why economic agents has to prefer to pay higher and higher prices for a depleting resource just to sustain consumption and production a bit higher for a while. Quite the reverse, rational agents would design a smooth contraction path and prefer to a pay a constant price to resource in the steady state.

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Annex A

Recall that equation (19) is

$$K(t) = \frac{a \cdot (q(0))^{-\frac{1}{\theta}} c(0)}{\left[\frac{\rho}{\theta} + \frac{(1-\alpha)\delta}{\alpha}\right]} \left(a + b * e^{-\frac{1-\alpha}{\alpha}\delta} \right)^{\frac{1}{(1-\alpha)}} e^{-\frac{\rho}{\theta}t}$$

$$+ \frac{a \cdot (q(0))^{-\frac{1}{\theta}} c(0)}{\left[\frac{\rho}{\theta} + \frac{2(1-\alpha)\delta}{\alpha}\right]} \left(a + b * e^{-\frac{1-\alpha}{\alpha}\delta} \right)^{\frac{1}{(1-\alpha)}} e^{-\frac{\rho}{\theta}t} e^{-\frac{(1-\alpha)\delta}{\alpha}t} \qquad \text{(A.1)}$$

where $a = \frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta}$ and $b = \left((q(0))^{1-\alpha} - \frac{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}{\delta} \right)$. Given that $K(0) = K_0$, (A.1) yields

$$K_0 = \frac{a \cdot (q(0))^{-\frac{1}{\theta}} c(0)}{\left[\frac{\rho}{\theta} + \frac{(1-\alpha)\delta}{\alpha}\right]} (a+b)^{\frac{1}{(1-\alpha)}} + \frac{a \cdot (q(0))^{-\frac{1}{\theta}} c(0)}{\left[\frac{\rho}{\theta} + \frac{2(1-\alpha)\delta}{\alpha}\right]} (a+b)^{\frac{1}{(1-\alpha)}} \qquad \text{(A.2)}$$

The two unknowns in (A.2) are $q(0)$ and $C(0)$. The second equation for solving these two unknowns comes from resource constraint equation, $S_0 = \int_0^\infty R(t)dt$. After simplification, we find these equations as follows:

$$K_0 = (q(0))^{-\frac{1}{\theta}} c(0)\alpha\theta \left[\frac{a}{\rho\alpha + \theta(1-\alpha)\delta} + \frac{b}{\rho\alpha + 2\theta(1-\alpha)\delta} \right] \qquad \text{(A.3)}$$

$$S_0 = (1-\alpha)^{\frac{1}{\alpha}} \theta (q(0))^{-\frac{1}{\theta}} c(0)\alpha\theta \left[\frac{1}{\rho} \frac{a}{\rho\alpha + \theta(1-\alpha)\delta} + \frac{a}{\rho\alpha + \theta(1-\alpha)\delta} \frac{b}{\rho\alpha + 2\theta(1-\alpha)\delta} \right] \qquad \text{(A.4)}$$

Annex B

The Dasgupta and Heal (1974) model implies the following results for $\chi = K/R$, r, q, and C:

$$\chi(t) = \left((1-\alpha)t + (\chi(0))^{1-\alpha}\right)^{1/(1-\alpha)} \tag{B.1}$$

$$r(t) = \frac{\alpha}{(1-\alpha)t + (\chi(0))^{1-\alpha}} \tag{B.2}$$

$$q(t) = (1-\alpha)\left((1-\alpha)t + (\chi(0))^{1-\alpha}\right)^{\alpha/(1-\alpha)} \tag{B.3}$$

$$C(t) = \left((1-\alpha)t + (\chi(0))^{1-\alpha}\right)^{\frac{\alpha}{\theta(1-\alpha)}} e^{-\frac{\rho}{\theta}t} e^{c_1} \tag{B.4}$$

For solving K, we follow the same procedure. First, recall that the capital accumulation function is

$$\dot{K} - \left(\frac{r(t)}{\alpha}\right)K = -C(t) \tag{B.5}$$

First, let us define the integrating factor $\pi(t) = e^{\int P(t)dt}$, where $P(t) = -\left(\frac{r(t)}{\alpha}\right)$. The solution of this integration yields that $\pi(t) = u^{-\frac{1}{1-\alpha}}$, where $u = ((1-\alpha)t + (\chi(0))^{1-\alpha})$. Next, multiply both sides of (B.5) by the integrating factor $\pi(t)$, which allows one to re-formulate the problem as follows:

$$K(t) * u^{-\frac{1}{1-\alpha}} = -e^{c_1} \int \left((1-\alpha)t + (\chi(0))^{1-\alpha}\right)^{\frac{\alpha-\theta}{\theta(1-\alpha)}} e^{-\frac{\rho}{\theta}t} dt + c_2 \tag{B.6}$$

It is not possible to solve the integration in (B.6) analytically, unless a specific value is assigned for θ. Suppose that $\theta = \frac{\alpha}{2-\alpha}$. Then, K is found as:

$$K(t) = e^{c_1} u^{\frac{1}{1-\alpha}} e^{-\frac{\rho}{\theta}t} \left((1-\alpha)t - 1 + (\chi(0))^{1-\alpha}\right) + c_2 u^{\frac{1}{1-\alpha}} \tag{B.7}$$

Transversality condition that $\lim_{t\to\infty}[\mu(t)\cdot K(t)] = 0$ implies that c_2 must be zero. Hence, we found $K(t)$ as

$$K(t) = e^{c_1} (\theta/\rho) u^{\frac{1}{1-\alpha}} e^{-\frac{\rho}{\theta}t} \left((1-\alpha)t - 1 + (\chi(0))^{1-\alpha}\right) \tag{B.7`}$$

Notably, the term in the parenthesis on the right hand side can be negative for low values of t and $\chi(0)$. Therefore, we need to assume that $\left((1-\alpha)t - 1 + (\chi(0))^{1-\alpha}\right) > 0$. Given the fact that this economy is depleting over time, $\chi(0) > 1$ is not a strong assumption. The value of R can be found by using profit maximizing factor employment conditions of K and R. It follows that

$$R(t) = e^{c_1}(\theta / \rho) e^{-\frac{\rho}{\theta}t}\left((1-\alpha)t - 1 + (\chi(0))^{1-\alpha}\right) \tag{B.8}$$

Finally, from the resource constraint, we find that

$$S_0 = e^{c_1}(\theta / \rho)^2\left((1-\alpha)((\theta / \rho) - 1) + ((\chi(0))^{1-\alpha} - 1)\right) \tag{B.9}$$

Equations (B.7\`) and (B.9) determine values of $R(0)$ and $C(0)$. Hence, full solution is found.